

Nonlinear excitation control of multimachine systems via the invariant-set design

Hisham M. Soliman¹, Ehab H. E. Bayoumi², Farag Ali El-Sheikhi³, Fawzan Salem⁴

¹Department of Electrical Power Engineering, Faculty of Engineering, Cairo University, Cairo, Egypt

²Department of Mechanical Engineering, Faculty of Engineering, The British University in Egypt, El Sherouk, Egypt

³Department of Electrical and Electronics Engineering, Faculty of Engineering and Architecture, Istanbul Esenyurt University, Istanbul, Turkey

⁴Department of Power Electronics and Energy Conversion, Electronics Research Institute, Cairo, Egypt

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ABSTRACT

Power grids are inherently vulnerable to many uncertainties. All power networks are prone to instability because of the uncertainties inherent in the operation of power systems. Rotor-angle instability is a challenging issue, and if not properly managed, could give rise to cascading failures and even blackouts. This paper addresses the generator excitation system's state feedback sliding mode control (SMC). The global system is divided into multiple subsystems to achieve decentralized control. A disturbance is defined as the influence of the system as a whole on a specific subsystem. The state-feedback controller is to be designed taking into account the disturbance attenuation level, ensuring the closed-loop system's asymptotic stability. The SMC designing algorithm is described; it is based on precisely determining the sliding surface utilizing the invariant-set (ellipsoid) technique. The control structure ensures that mismatched disturbances in power systems have little impact on the system trajectory in the sliding mode. Moreover, the proposed controllers are represented in this paper using linear matrix inequalities (LMIs) and the Lyapunov theory approach. Finally, a multi-machine model is implemented to demonstrate the success of the suggested approach, and a comparison between the proposed SMC and the conventional one demonstrates its superiority.

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Corresponding Author:

Fawzan Salem

Department of Power Electronics and Energy Conversion, Electronics Research Institute

Cairo, Egypt

Email: fawzan@eri.sci.eg

1. INTRODUCTION

- A review of associated works

Ensuring the security and reliability of interconnected power networks is one of the most difficult challenges [1]. Three types of instability might arise during the functioning of a power system: rotor-angle instability, voltage instability, and instability in frequency [1]. Load frequency control with practical restrictions on generation rate and magnitude is illustrated [2]. Instability in rotor angle happens if the discrepancy in rotor angle among separated generating units in a system surpasses an acceptable limit because of unwanted transient events (uncertainties). This could result in increasing power oscillations and eventually cause the system's entire collapse [3]. Rotor angle stability must be ensured to maintain the synchronism of synchronous generators.

The local-machine oscillation is the most common among all types of power system oscillations. These oscillations occur if one or more generators swing against the whole power grid (0.7-3 Hz). Another

type of power system oscillation is called an inter-area oscillation. This oscillation occurs when one group of synchronous generators in a certain power zone swings against another group of generators in a different zone (0.1-0.7 Hz) [1], [3].

The disturbance will cause oscillations, which could have a significant effect on the stability of a power system, concerning small and large signal (perturbation) stability. Synchronous generators have an automatic voltage regulator (AVR) that adjusts the generator's terminal voltage [1], [3]. The excitation channel loop high gain enhances the steady state error, although it could introduce instability. System oscillations can be damped if the AVR is properly adjusted. For system oscillations reduction, the excitation channel could be supplemented with another stabilizing signal, through a power system stabilizer PSS [1], [3]. Numerous research activities have been conducted to determine the extent to which the PSS and AVR can contribute significantly to power system stability. The PSS and AVR essentially compete with one another. The fast response and high gain of the AVR reduce oscillation stability of a power system and, at the same time, increase transient stability, and vice versa. In contrast, the PSS diminishes transient stability by overriding the voltage signal to the exciter while improving oscillation stability [4]. AVR and PSS designs (both dynamically linked) can, however, be coupled to deliver maximum power stability for oscillation and transient stability studies [4].

System uncertainty might be either stochastic or deterministic. The following are techniques for PSS/AVR systems with deterministic uncertainty design: i) PSS is widely utilized nowadays. Its architecture could be a proportional-integral-derivative (PID) [5], a multi-band [6], or a single- or double-stage lead-lag with one or two inputs; ii) Among resiliency techniques employed include Kharitonov Theory [7], qualitative feedback theory (QFT) [8], sliding mode control (SMC) [9], and H_∞ for disturbance attenuation [10]; iii) PSS with adaptive capabilities [11]; and iv) Intelligent-based PSS employs a variety of evolutionary techniques, including particle swarm optimization (PSO), tabu search, genetic algorithm (GA), simulated annealing [12], neural networks [13], fuzzy logic [14], and adaptive fuzzy SMC [15].

Among the various strategies mentioned above, SMC is one of the most successful nonlinear control methods for stabilizing nonlinear power systems because of its exceptional resistance to uncertainty. The preceding paragraph concerns power systems with deterministic uncertainties. In what follows, we shall present the research efforts in stabilization of power systems subject to stochastic (or probabilistic) uncertainties. Because of the external stochastic disturbance, abrupt and sharp slope changes, ordinary differential equations (ODEs) cannot be used to model power system stochastic stability. Stochastic differential equations are used to represent system dynamics in the presence of random swings. There has been little research activity studying stochastic stability in power systems [16], [17]. The following are examples of stochastic disturbances occurring in a power system:

- a) Lightning strikes hitting a transmission line can cause random topological changes in the system (leading to the opening and closing of circuit breakers in power systems).
- b) Random wind speed fluctuations affect line reactance, as it is based on phase conductor spacing is modelled as a stochastic external disturbance.
- c) The line resistance is a function of environmental temperature changes/fluctuations.

Markov jump can be used to simulate such stochastic disturbances and random topological changes. The observer-based or state excitation feedback control utilizing an invariant ellipsoid approach is described in [18], [19]. Poznyak *et al.* [20] investigate power systems' stochastic stability with time delay.

Power networks can be stabilized by centralized or decentralized control. Although regulating large power networks with a single hub computer (centralized control) gives good control grip, the following limitations apply: i) In case the hub fails, there will be a complete loss of control; ii) Transmitting the states to the hub requires a costly communication network; iii) Data packet loss and time delays impair system performance and might lead to instability; and iv) To address these restrictions, this study proposes the following taking into account the challenges encountered when implementing decentralized stabilization in a power system with multiple machines.

- Contributions

- i) Control decentralization is accomplished by segmenting the multi-machine system into smaller subsystems. A controller that only makes use of the local states is included with every machine. The rest of the system's effect is seen by a specific machine as an exogenous disturbance that needs to be lessened.
- ii) For excitation control of the power system, the SMC design follows the procedure outlined in [9]. It is based on employing the invariant ellipsoid approach to accurately select the sliding surface.
- iii) Methodology in [9] is extended, in this paper, to tackle the control decentralization.
- iv) The proposed SMC guarantees minimizing the impact of unmatched disturbances on system performance. This is unlike traditional SMCs, which cannot deal with unmatched disturbances.
- v) A straightforward design procedure that requires no costly computational methods.
- vi) Lyapunov functions are used to assure closed-loop stability.

vii) The proposed controller is tested using a multi-machine system. The simulation studies conducted on that system confirm the theoretical work presented in this paper.

- Notations

This work utilized the conventional notation as follows: $(\cdot)'$ represents a matrix or a vector transpose. Definite positive and negative symmetric matrices are symbolized by $P > 0$ and $P < 0$, respectively. $(M + N + M' + N')$ notation is represented by $(M + N + *)$. A symbol $(*)$ in any matrix means the symmetric part; for example $\begin{bmatrix} M & N \\ * & Z \end{bmatrix}$ indicates $\begin{bmatrix} M & N \\ N' & Z \end{bmatrix}$. I and 0 symbols stand for identity and zero matrices, respectively, $\dot{x}$ is the x time derivative.

The paper is organized as follows: i) Section 1 discusses previous studies on power system stabilization; ii) Section 2 presents a multi-machine system modelling and problem description; iii) Section 3 demonstrates the invariant ellipsoid method for state feedback sliding mode control design; iv) The results of a benchmark example simulation are detailed in section 4; and v) Conclusions are summarized in section 5.

2. MULTI-MACHINE MODELING AND PROBLEM STATEMENT

Consider the following power system, which consists of $N+1$ machines. The machine number $N+1$ is used as a reference. A third-order dynamic model is used to represent each machine [1]:

$$\begin{aligned}\dot{\delta}_i &= w_i \Delta w_i, \Delta w_i = (w_i - w_0) \\ \dot{w}_i &= \frac{1}{2H_i} (P_{mi} - P_{ei}) \\ \dot{E}'_{qi} &= \frac{1}{T'_{do}} (E_{fi} - E_{qi})\end{aligned}\quad (1)$$

where

$$\begin{aligned}E_{qi} &= E'_{qi} + (x_{di} - x'_{di})I_{di} \\ E_{fi} &= u_i \\ P_{ei} &= \sum_{j=1}^n E'_{qi} E'_{qj} B_{ij} \sin(\delta_i - \delta_j) \\ Q_{ei} &= - \sum_{j=1}^n E'_{qi} E'_{qj} B_{ij} \cos(\delta_i - \delta_j) \\ I_{di} &= - \sum_{j=1}^n E'_{qj} B_{ij} \cos(\delta_i - \delta_j) \\ I_{qi} &= \sum_{j=1}^n E'_{qj} B_{ij} \sin(\delta_i - \delta_j) \\ E_{qi} &= V_{ti} + \frac{Q_{ei} x_{di}}{V_{ti}}\end{aligned}\quad (2)$$

The multi-machine power system model notations are provided in the above list of symbols. Assume that machine # i is working at an operating point $\delta_{i0}, w_{i0}, P_{ei0}$. Assume $P_{mi} = \text{constant}$, $\Delta\delta_i = \delta_i - \delta_{i0}$ and $\Delta w_i = w_i - w_{i0}$. The dynamics of the rest of the N machines for small oscillations around the operating point can be represented by the linearized state equation.

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u}\quad (3)$$

The selected test system is the three-machine power system [1] shown in Figure 1.

Each machine's state vector (x) and control input (u) are $[\Delta\delta \quad \Delta w \quad \Delta E'_q]'$, and E_f respectively. Using machine # 3 as a reference for the benchmark system $N+1=3$, the dynamics of the other two machines, the (3) becomes $\hat{x} = [x'_1 \quad x'_2]'$, similarly for $\hat{u} = [u_1 \quad u_2]'$.

$$\text{The matrices } \hat{A} = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix}, \hat{B} = \text{block diag} [B_1 \quad B_2].$$

The above matrices' numerical values are provided in the Appendix. System (3) is divided into three subsystems to achieve control decentralization, with the dynamics for each machine being:

$$\dot{x} = Ax + Bu + Dw, x(0) = x_0, w = \hat{x},\quad (4)$$

It is important to note that the influence of the rest on a specific subsystem is treated as an external disturbance. Its impact should be lessened. Regarding any generator i , where $i=1,2,\dots,N$, and $N=2$ for our system.

$$A = A_i, B = B_i, D = D_i = [A_{i1} \ A_{i2} \ \dots \ 0_{ii} \ \dots \ A_{iN}], w = \hat{x}$$

For every generator, $x(t)$ is the state of n dimensions, $u(t)$ represents the control input of dimension m , and $w(t)$ denotes the external disturbance of dimension k ($k=n.N$). The (A, B) pair is controllable. The external disturbance is not known but bound as:

$$w' Q_w w \leq w_0 + x' Q_x x \quad (5)$$

Q_w, Q_x , the positive definite matrices, as well as the positive scalar w_0 are given.

Power systems external disturbance is mismatched as the D matrix is not in the range of matrix B , which means that $D \neq B\Phi$, where $\Phi =$ any matrix. Each machine's decentralized state feedback control utilizes its own local states is:

$$u = Kx \quad (6)$$

In order to maintain the stability of the benchmark system while adhering to constraint (5), the controller gain K should be accordingly designed. For the controller (6), the SMC is selected due to its remarkable resilience to the system's uncertainties.

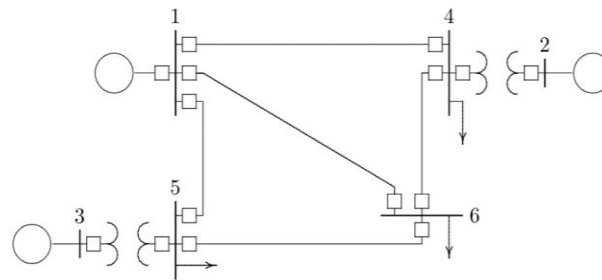


Figure 1. Multi-machine 6-bus test power system

3. THE INVARIANT ELLIPSOID APPROACH IN SMC

The invariant ellipsoid approach is a new and effective control mechanism [9]. The approach stabilizes linear and nonlinear systems against plant uncertainty and mitigates the impact of external disturbances. This approach has various applications, including microgrid control [21]. The SMC structure to stabilize disturbed systems is [22]:

$$u(t) = -(\tilde{C}B)^{-1}\tilde{C}Ax(t) - M(x(t)).\text{sign}[\tilde{C}x(t)], \quad M(x(t)) > 0 \quad (7)$$

The matrix $\tilde{C} \in \mathbb{R}^{m \times n}$ represents a sliding surface with $\det(\tilde{C}B) \neq 0$. $M(x)$, the positive control gain function, takes the form:

$$M(x) = \sqrt{(\alpha + x'Rx)}, \quad (8)$$

The positive definite matrix R and the scalar α are bounded parameters because $\alpha < \alpha_{\max}$, $\|R\| \leq \beta$, α, β are scalars > 0 . If the disturbance is inside the range (B) , also known as the matched condition, this control can eradicate it.

The matching condition in power systems does not hold. Note that the power system dynamics (2) have "unmatched disturbances" that cannot be suppressed by any conventional sliding-mode control. Therefore, the greatest obstacle is to develop a sliding-mode control which employs the invariant-ellipsoid technique to mitigate, in some sense, mismatched disturbance effects [9].

3.1. Invariant ellipsoid technique [9]

Definition 1: Ellipsoid $E(P)$ is as follows:

$$E(P) = \{x'P^{-1}x < 1\}, P > 0, \quad (9)$$

This ellipsoid, centered at the origin with a configuration matrix P , is state invariant for the system (4) concerning disturbance (5) if any system state trajectory starting inside the ellipsoid stays inside it for all time instants $t > 0$. This property guarantees that the system's state remains bound within a specific region despite the presence of external disturbances. However, if a trajectory begins outside the ellipsoid and becomes attracted to it over time, the ellipsoid is also known as attractive.

The invariant ellipsoid can be thought of as an example of how mismatched perturbations (5) impact the system (4). In the SMC application, the smallest, in a sense, invariant ellipsoid (9) provides a "minimum deviation of any possible trajectory from the origin in a sliding mode". The main challenge is to develop a control configuration that allows any system (4) trajectory to converge into the beforehand specified "minimum invariant ellipsoid". For such optimization issues, the typical criterion is minimal $\text{str}(P^{-1})$. The matrix P^{-1} trace represents the sum of the squares of the ellipsoid's semi-axes. Using linear matrix inequalities optimization, the invariant ellipsoid of the linear disturbed control system is utilized to develop an appropriate sliding surface $\tilde{C}x = 0$, [9].

3.2. System decomposition and main results

Given rank $(B) = m$, matrix B can be decomposed as follows:

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

B_1 , and B_2 have dimensions of $(n-m), m$ and m, m respectively and $\det(B_2) \neq 0$. The nonsingular transformation is present in this situation [23].

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = Gx, \text{ where } G = \begin{bmatrix} I_{n-m} & -B_1B_2^{-1} \\ 0 & B_2^{-1} \end{bmatrix}$$

System (4) can then be reduced to the form:

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}x_2 + D_1w \\ \dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + u + D_2w \end{aligned} \quad (10)$$

where $n-m$, and m are the dimensions of x_1 , x_2 , respectively. Matrices A_{11}, A_{22} have proper dimensions and should not be confused with those listed in the Appendix. Therefore,

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = GAG^{-1} \text{ and } \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = GD$$

where $(n-m), k$, and m, k are the dimensions of D_1 , D_2 , respectively. It must be noted that if the system (4) is controllable, then $\{A_{11}, A_{12}\}$ is controllable as well [23]. The following theorem is derived in [9].

Theorem: the solution $(\alpha, \tau, \delta, L, Y, Z, Z)$ of the minimization problem

$$\text{minimize trace}(Z)$$

Subject to the following inequality constraints:

$$\begin{aligned} &\begin{bmatrix} \beta^2 I_n & * \\ R & I_n \end{bmatrix} \geq 0, R \geq \frac{\alpha}{w_0} Q_x, \\ &0 < \alpha \leq \alpha_{\max}, \\ &\begin{bmatrix} (A_{11}X - A_{12}Y + *) + \tau X & * & * \\ D'_1 & -\frac{\tau}{w_0} Q_w & * \\ \begin{bmatrix} X \\ -Y \end{bmatrix} & 0 & -\frac{w_0}{\tau} Q_x^{-1} \end{bmatrix} < 0, \end{aligned}$$

$$\begin{bmatrix} X & * \\ \begin{bmatrix} X \\ -Y \end{bmatrix} & GZG' \end{bmatrix} \geq 0, \begin{bmatrix} \frac{\alpha}{w_0} Q_w & * \\ GD & \begin{bmatrix} \delta X & * \\ 0 & L \end{bmatrix} \end{bmatrix} > 0, \begin{bmatrix} \frac{1}{\delta} X & * \\ Y & I_m - L \end{bmatrix} \geq 0$$

gives the minimal invariant ellipsoid of the system (4). Furthermore, the SMC as provided by (7) and (8), with the sliding surface, is $\tilde{C} = (YX^{-1}, I_m)G$. Using any gradient-free approach such as particle swarm optimization technique, the aforementioned minimization problem can be handled. Proof: see [9].

Solving the aforementioned theorem under assumptions ($\alpha_{\max} = 100, \beta_{\max} = 7, Q_{w0} = I, Q_x = I, w_0 = 0.1$) yields a decentralized excitation control system. This system is designed using the sliding surfaces presented in Table 1, and the scalar sliding mode controller, $u(t)$, for each machine takes the form specified in (7). The parameters, including the constants α, R and the Sliding surface $\tilde{C}$, are comprehensively illustrated in Table 1 to provide all necessary implementation details.

Table 1. Proposed SMC

Machine #	α	R	Sliding Surface $\tilde{C}$
1	0.69845	[6.9922, 1.5281e-14, -6.1398e-14; 1.5281e-14, 6.9922, -2.1097e-13; -6.1398e-14, -2.1097e-13, 6.9922]	[-2.539, -196.64, 5.8005]
2	0.69936	[6.9968, -1.5572e-14, 2.6983e-14; -1.5572e-14, 6.9968, 1.4594e-14; 2.6983e-14, 1.4594e-14, 6.9968]	[-2.4757, -270.33, 5.9988]

4. SIMULATION VERIFICATION

The simulation findings demonstrate the proposed approach's robustness and performance in comparison with a classical sliding mode excitation control. Simulation results were implemented using MATLAB LMI Toolbox [24], Yalmip Interface [25], and Sedumi Solvers [26]. The Ode23 solver was used by a tolerance of 10^{-3} . To eliminate the chattering problem, the sign function in (7) is replaced by a saturation function [23].

4.1. The multi-machine test power system simulation

A three-phase fault is applied at Generator 1 to confirm stability enhancement due to the proposed stabilizer. This leads to rotor angle deviation $x_0 = [0.3, 0, 0, 0.1, 0, 0]'$. Three different operating scenarios were investigated: nominal, light, and heavy loads. After clearing the fault. The controller supports the system in quickly recovering to a steady operational condition. Figure 2 illustrates that, regarding the damping effect and settling time at nominal load, the suggested technique outperforms the traditional SMC. Moreover, when compared to the standard SMC, the suggested SMC generates oscillations with smaller magnitudes and frequencies. As a result, the generator rotor's useful life is extended and fatigue is reduced.

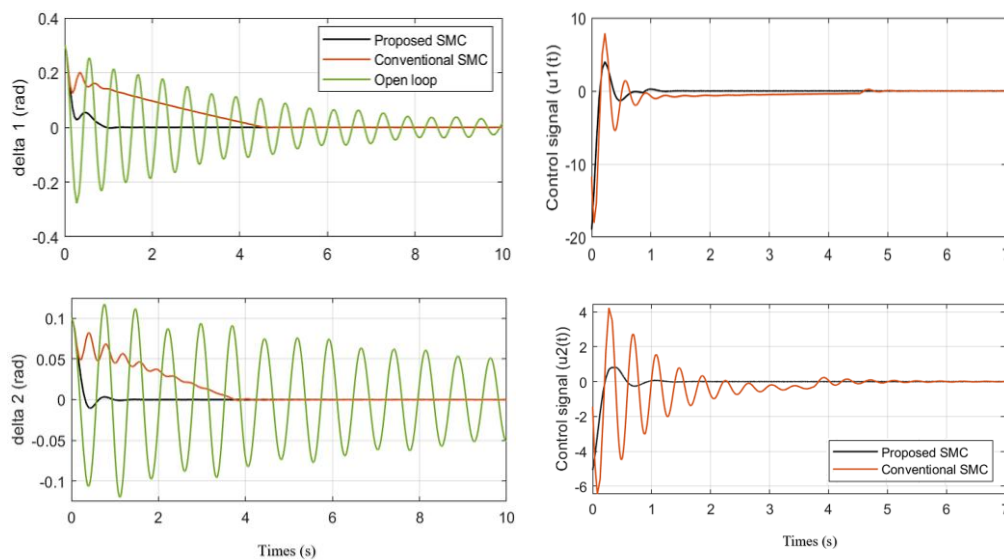


Figure 2. Rotor-angle variations and control-input under nominal load condition: (a) delta 1 (rad), (b) control signal 1 ($u_1(t)$), (c) delta 2 (rad), and (d) control signal 2 ($u_2(t)$)

4.2. Robustness assessment

The proposed stabilizer's resiliency is further evaluated by testing the power system under both heavy and light load conditions, as shown in Figures 3 and 4. These tests are crucial for assessing the controller's robustness across a wide range of operational scenarios. The results consistently demonstrate that the proposed controller outperforms its traditional counterpart in the multi-machine power system.

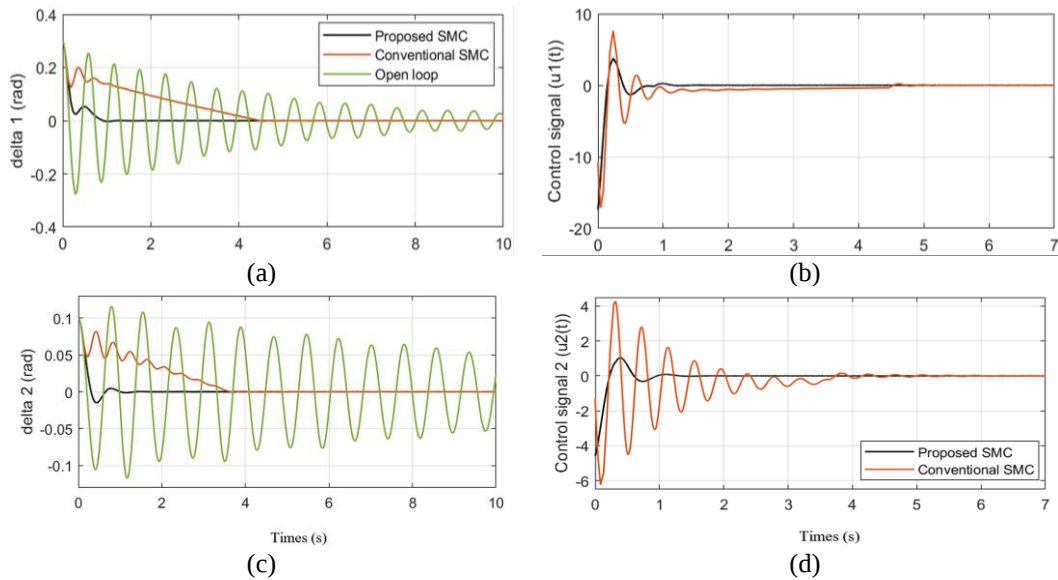


Figure 3. Rotor-angle variations and control-input under high load condition: (a) delta 1 (rad), (b) control signal 1 ($u_1(t)$), (c) delta 2 (rad), and (d) control signal 2 ($u_2(t)$)

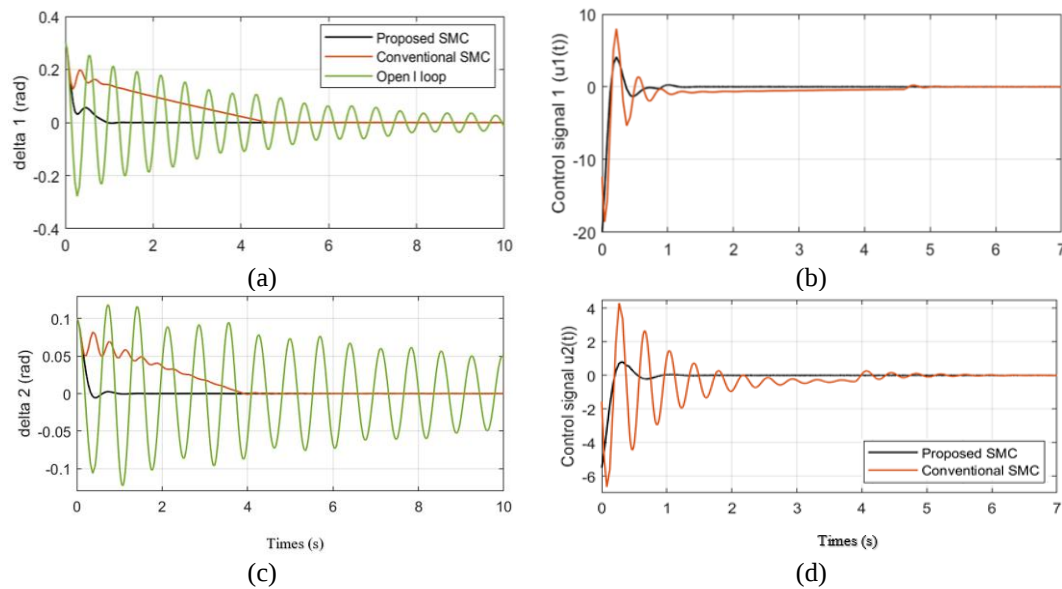


Figure 4. Rotor-angle variations and control input under light load conditions: (a) delta 1 (rad), (b) control signal 1 ($u_1(t)$), (c) delta 2 (rad), and (d) control signal 2 ($u_2(t)$)

In this study, we employ a diverse range of operating points to illustrate the efficiency of the proposed control for oscillation damping following a large disturbance on a power system. This comprehensive approach ensures that the controller's performance is robustly evaluated under various real-world conditions. The results consistently show that the proposed controller provides enhanced transient responsiveness and greater robustness compared to conventional SMC.

5. CONCLUSION

This paper introduces a new technique for power system decentralized excitation control. Decentralization of excitation control is performed by dividing the entire system into smaller systems and placing a controller on each generator that utilizes its local states. The rest of the system's impact on a specific generator is regarded as external disturbances that should be mitigated. To reduce the mismatched external disturbances, the proposed design is based on SMC and employs the invariant ellipsoid technique. Simulations of a multi-machine test system illustrate the efficacy of the proposed control method. Various loading scenarios are used to assess the proposed decentralized excitation SMC efficiency. This clearly indicates the superior performance of the proposed technique over conventional SMC.

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AUTHOR CONTRIBUTIONS STATEMENT

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Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Hisham M. Soliman	✓	✓		✓	✓	✓		✓	✓	✓		✓	✓	
Ehab H. E. Bayoumi		✓	✓	✓	✓	✓		✓	✓		✓	✓		
Farag Ali El-Sheikhi	✓			✓		✓	✓			✓				
Fawzan Salem		✓		✓		✓	✓			✓	✓			✓

C : **C**onceptualization

M : **M**ethodology

So : **S**oftware

Va : **V**alidation

Fo : **F**ormal analysis

I : **I**nvestigation

R : **R**esources

D : **D**ata Curation

O : Writing - **O**riginal Draft

E : Writing - Review & **E**diting

Vi : **V**isualization

Su : **S**upervision

P : **P**roject administration

Fu : **F**unding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest and the paper is not published or submitted anywhere.

DATA AVAILABILITY

Data availability is not applicable to this paper as data used or analyzed are given in this paper.

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APPENDIX

- The multi-machine model

The benchmark state equations under nominal load are as follows:

$$A = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix} = \begin{bmatrix} 0 & 377 & 0 & 0 & 0 & 0 \\ -0.335 & 0 & -0.3474 & 0.01158 & 0 & 0.004632 \\ -0.4637 & 0 & -0.8703 & 0.02048 & 0 & 0.02754 \\ 0 & 0 & 0 & 0 & 377 & 0 \\ 0.00851 & 0 & 0.003724 & -0.1919 & 0 & -0.1834 \\ 0.01223 & 0 & 0.01686 & -0.1786 & 0 & -0.4645 \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.1724 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.1667 \end{bmatrix}^T$$

which can be decomposed into two subsystems as follows:

$$\dot{x}_1 = A_1 x_1 + B_1 u_1 + D_1 w, D_1 = [0, A_{12}], w = \hat{x}$$

$$\dot{x}_2 = A_2 x_2 + B_2 u_2 + D_2 w, D_2 = [A_{21}, 0], w = \hat{x}$$

BIOGRAPHIES OF AUTHORS



Hisham M. Soliman    received the B.Sc. (Hons.) and M.Sc. degrees in electrical engineering from Cairo University, Cairo, Egypt, in 1972 and 1975, respectively, and the Ph.D. degree in automatic control from Paul Sabatier University, Toulouse, France, in 1980. He taught at the University of Saskatchewan, Canada; Garyounis University, Libya; United Arab Emirates University (UAEU), United Arab Emirates; Yanbu Industrial College, Saudi Arabia; and Sultan Qaboos University, Oman. He is currently a professor of electrical engineering at Cairo University. He can be contacted at email: mohammedhisham@eng.cu.edu.eg.



Ehab H. E. Bayoumi    received the B.Sc. degree in electrical power engineering from Helwan University, Egypt, in 1988, the M.Sc. degree in electrical power engineering from Ain Shams University, Egypt, in 1996, and the Ph.D. degree in electrical power engineering from Cairo University, Egypt, in 2001. He has been with the Electronic Research Institute (ERI), Cairo, Egypt, since 1990. From 2000 to 2001, he was a Visiting Researcher at LUT, Finland. He was appointed an assistant professor at Chalmers University of Technology, Sweden, from 2003 to 2005. He was appointed as an associate professor with ERI in 2008. In 2014, he was retained as a full professor with ERI. He additionally served in positions at Yanbu Industrial College, Saudi Arabia; the Higher Colleges of Technology, United Arab Emirates; the University of Eswatini, Eswatini; The Egyptian Chinese University, Egypt; The University of Botswana and the British University in Egypt. He is currently a full Professor at British University in Egypt. His research interests include high-performance AC machines, power quality, switching power converters in smart and micro-grids, and nonlinear control applications in power electronics, smart grids, and electric drive systems. From 2013 to 2022, he was appointed the Editor-in-Chief of the International Journal of Industrial Electronics and Drives. He can be contacted at email: ehab.bayoumi@gmail.com.



Farag Ali El-Sheikhi    received the M.Sc. degree in the area of power system simulation from the University of Missouri, Columbia, USA, and the Ph.D. degree in the area of power systems reliability evaluation from the University of Saskatchewan, Canada. He was a professor of electrical energy and power system reliability at the University of Benghazi, Libya. He is currently a teaching staff member with the Electrical and Electronics Engineering Department, Faculty of Engineering and Architecture, Istanbul Esenyurt University, Istanbul, Turkey. His research interests include power systems reliability evaluation, power generation systems, transmission and distribution networks planning, power networks, and industrial systems operation and maintenance, where he has many publications in these areas. He can be contacted at email: farag.elsheikhi@gmail.com.



Fawzan Salem    is an associate professor at Electronics Research Institute, Power Electronics and Energy Conversion Department, Egypt. He received a B.S. from Zagazig University, an M.Sc. and Ph.D. from Cairo University in 1992, 1998 and 2006, respectively. Worked as a full-time assistant professor between 2009 and 2020 at Yanbu Industrial College, Saudi Arabia. His research interests include intelligent control techniques, renewable energy, and optimization algorithms. He can be contacted at email: fawzan@eri.sci.eg.